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ABSTRACT

This booklet was prepared to provide elementary school teachers with background information and strategies for teaching remedial mathematics. Individual sections focus on educational theory and mathematics, which outlines associationist and developmental theories, summarizes traditional and contemporary teaching styles, and relates theories to practice; diagnostic and prescriptive mathematics, which notes reasons for failure and several types of remedial approaches; teacher attitude and classroom climate, which focuses on ideas for improving each; attitudes and mathematics, which indicates how attitudes are formed and measured; language and mathematics, which includes information on language development and problems; socioeconomic variables and mathematics, which briefly notes their role; and sex differences and mathematics, which states some differences between the achievement of boys and girls. Brief case studies are included with most sections. (MNS)

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STRATEGIES FOR TEACHING REMEDIAL MATHEMATICS

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STRATEGIES
FOR TEACHING
REMEDIAL MATHEMATICS

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Extra copies of this booklet, each priced at (includes postage within Australia), are avialable from....

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INTRODUCTION

This booklet is the result of part of the requirements of the subject The Remedial Teaching of Primary School Mathematics taught at the Albury Wodonga campus of Riverina C.A.E. in 1980.
The lecturer in charge of the subject wrote the initial sections of the booklet, and has wielded a sometimes formidable editorial pen over the contents of the remainder which were initially written by the students enrolled in the subject:

Glenda Burke
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Cathy O'Shanassy
Ian Prenter
Lorna Read
Gudrun Reid
Ruth Shanahan
Herwig Waldhuber
Frances Walsh
Carmel Wilson
Christine Woolner
Julie Wright

All the above students are experienced teachers who were studying part-time. This booklet is written in a style we hope is useful for school teachers. We would appreciate any comments you would care to make, especially concerning possible improvements to a later version of the booklet.

Neil Hall
November, 1980.

EDUCATIONAL THEORY AND MATHEMATICS

LEARNING MATHEMATICS

The large number of views as to how the learning of mathematics takes place may be roughly placed into two categories: associationist theories and developmental theories.

Associationist Theories

The first associationist theory was Watson's Behaviourism, dating from the early part of this century. The fundamental assertion of this theory is that learning occurs through a response to a given stimulus.

About the same time Thorndike produced his theory of Connectionism. This theory is based on similar grounds as Watson's behaviourism: learning occurs through responses to given stimuli. Thorndike argued that the strength of these neural connections could be increased by exercise and reinforcement .... this he called the Law of Effect.

That is, the Law of Effect states that if new learning takes place that learning will best be remembered if it is practised and positively reinforced. Thorndike argued, too, that experiences would not be remembered if they were negatively reinforced ... that is, if children behaved badly and were punished, this behaviour should occur less frequently. Neither Thorndike

nor any other researcher has been able to provide empirical evidence that the latter section of the Law is true. Data is readily available to support the practice and positive reinforcement section of the Law, so Thorndike altered his views to take into account these facts and defined the Truncated Law of Effect.

In the 1940's Skinner wrote his theory of Operant Conditioning, a theory that was to be the most influential of all the associationist theories. Operant conditioning is concerned with the reinforcement of observed behaviour: the stimulus is virtually irrelevant, it is the response that is all important. Skinner argued that reinforcement increased the probability of a given behaviour occurring again, and this was so whether the reinforcement was pleasant or unpleasant: extinction of undesirable behaviour was possible only by ignoring that behaviour.

These associationist, or behaviourist, theories all imply that education is about altering students' behaviours, and cannot be concerned with anything else. Observed behaviour is the only way a teacher can know what a learner understands: implications from observed behaviour as to how the student is thinking are irrelevant according to these types of theories.

The implications of these theories for mathematics learning are that

mathematical concepts should be learned as stimulus response connections;
drill, practice and reinforcement are to be stressed;
understanding is irrelevant;
learners' individual differences are irrelevant;
children's existing cognitive development is irrelevant;
pre-requisite skills are unimportant.

Developmental Theories

In general, developmental theories argue the opposite case to the associationist theories. That is,

mathematical concepts should be learned through experience in mathematical ideas;

pratice is important, but reinforcement is much more intrinsic;
understanding is essential;
learners' individual differences, their existing skills and levels of cognitive development taken together indicate where the teacher starts from and implies that certain teaching methods and activities will be more relevant than possible alternatives.

The best known developmental model of learning is that of Piaget. Piaget argued that every individual proceeds through the same series of stages, in the same order, though at different rates and at differing ages: the type of thought of which the child is capable differs at each stage. These stages are

The sensorimotor stage (0 - 2 years)	where the child organises his world through his senses and actions. Language and thought processes are developing, but are minimal.
The preoperational stage (2 - 7 years)	here the child moves from an egocentric view of the world to early forms of social behaviour and conceptual thought. The child acquires the power to think but is influenced greatly by perception.
The concrete operational stage (7 - 15 years)	the child begins to think logically about the real world but is tied to concrete referrents and finds difficulty in generalising from one situation to another.
The formal operational stage (from 15 years)	now the child is able to think abstractly, he is able to deal with possibilities and hypotheses.

Piagetian theory asserts that learning occurs through the process referred to as adaptation. Adaptation is essential because the individual must adapt to his environment: to be in conflict with one's environment creates enormous pressures of anxiety and uncertainty. There are two aspects to the process of adaptation

assimilation	refers to the process of incorporating new experiences into one's present set of understandings;
accommodation	is the process of having to alter some aspect of one's present understandings because one has experienced an activity that does not fit in with what one already knows.

A primary school child may use the term "number sentence" to describe $5 + \square = 12$. When this child first sees statements such as $5 + x = 12$ he will assimilate these statements into his present understandings by referring to them as number sentences: shortly though, he will have to adjust his understandings to a new concept, he will have to accommodate the concept of "equations". A couple of years later, his concept of equations will have to alter once again: he will have to accommodate new types of equations, for example, quadratic equations.

Associationist theories take the correct answer given by the child as proof that sufficient learning has taken place. However, this is only a part of what mathematics education is about. In addition to learning a set of mathematical skills, contemporary mathematics educators would want children to understand the process that led to a solution, to express mathematical relationships in their own way and to experience the creative and aesthetic aspects of mathematics.

Piagetian theory provides solid foundations for the type of theory needed by mathematics educators. This theory helps us to see why certain mathematical topics cause children difficulty (their cognitive development is insufficient), gives an account of how children learn and so has implications as to how mathematics should be taught. Piaget's theory may be interpreted as supporting each of the following ideas

> modern mathematics programs are preferable to traditional mathematics programs because the former are more in line with how children think;
>
> mathematics teaching must involve actions by the child, and activity with objects: with formalisation delayed until sometime after the necessary notions have been acquired;
>
> mathematical material must be presented so as to encourage the child to re-discover the mathematics: and this can be interpreted in terms of both guided discovery teaching methods and allowing the child to express mathematical ideas in the way he finds most suitable.

TEACHING MATHEMATICS

There are many different approaches to the teaching of mathematics, but it is useful to look at the characteristics of those mathematics teaching styles that may broadly be referred to as traditional and contemporary.

TRADITIONAL

Teacher centred
the teacher is omnipotent.

Emphasis on teaching methods
Children are taught in a manner seen by the teacher to be most efficient.

Class teaching
Children are passive learners, they follow the teacher's instructions which have been addressed to the whole class.

External incentives
Stars, competitions, fear of failure.

Rigid procedures
Follow a given pattern to obtain the correct response.

Mainly arithmetic
Emphasis on content that is either right or wrong. Limited view of what constitutes mathematics.

Artificial sums
Any sums will do, simply to practice correct procedures.

Silence
Work by yourself, without talking. Discussion implies cheating.

CONTEMPORARY

Child centred
Children, their needs and interests are important.

Emphasis on learning methods
Children are taught according to how they learn most efficiently.

Group or individual learning
Children are active learners, each exploring mathematics in their own way.

Internally motivated
Curiosity, joy of discovery.

Flexibility of method
Children are encouraged to arrive at their own interpretation. There is a range of correct responses.

Wide range of mathematical topics
Mathematics is seen both as skills and knowledge to be learned and as activities to be experienced.

Real problem emphasis
Opportunity to explore real mathematical problems.

Discussion
Discussion to share experiences, both between children and between children and the teacher.

TRADITIONAL	CONTEMPORARY
Follow the textbook There is one, and only one, appropriate content and sequence in mathematics.	Needs of children The needs and interests of children, their previous experiences and readiness for particular types of work influence the choice of content.
Rote learning Emphasis on rules and rote learning in isolation from real experiences.	Understanding Rules are to be learned here, too: but they are taken from children's experiences with mathematical patterns and relationships.
Verbal and symbolic Mathematics is stressed as an abstract study, highly verbalised and symbolic.	More varied Mathematics involves activity and experience. Learning may be verbal and symbolic, but it also employs visual, kinaesthetic and concrete activities.
Ticks and crosses Passive acceptance of answers as either correct or incorrect.	Testing in concrete situation Am I correct? Children test their results.
Child as machine Children are a kind of machine, where "work" is all important.	Child as an individual Children are individuals. Mathematics contributes to the whole person; emotional factors are important.
Content Content is determined by the hierarchical nature of mathematics: that is, the child must adapt to the structure of mathematics.	Content Mathematical development is to be in accord with the child's cognitive development: that is, psychological bases of learning are relevant.
Rules Rules are basis of learning, and dominate all mathematics.	Exploration Rules are important, but are developed through exploration and enquiry.
Private work Children pursue their own work. Assistance, when necessary, must be sought from the teacher.	Deliberate communication Children communicate their experiences and discoveries to others by discussion and through graphs, note books, and so on.

In any table stressing dichotomies some generalisations are likely to be too broad. In general, the value judgements that underlie the traditional point of view are that mathematics is about facts, rules and right and wrong answers, that the structure of mathematics determines both the content and teaching methods to be used and that children are passive vessels into which mathematical knowledge must be poured. Contemporary mathematics education is underpinned by educational psychology, especially as related to how children learn: in addition to those views able to be proved empirically, contemporary ideas include the following value judgements - children are important and should be treated humanely, children's needs and interests are important, mathematics is more than just rules, mathematics must be experiential and mathematics must contribute to the broader goals of the school.

UNDERSTANDING and MATHEMATICS

Teachers often refer to the desirability of children understanding the work they do in school: but what really is meant by 'understanding'? In mathematics, is the achievement of the correct answer sufficient to infer that the child has understood, what else could be mean by 'understanding'? It is possible to distinguish between two types of understanding: instrumental understanding and relational understanding. Instrumental understanding refers to the child's ability to obtain the correct answer, the ability to follow a given procedure. Relational understanding requires more than instrumental understanding, it requires the child to know why the answer is correct and to know why a given procedure is relevant. The two types of understanding are described in more detail below.

INSTRUMENTAL	RELATIONAL
Understanding "how" Obtaining the correct answer implies understanding	Understanding "why" One must be able to explain why the answer is correct, and why the procedure is appropriate.
Easier to learn One learns many rules, and simply computes the values from these rules. The problem is to decide which rule is appropriate for a given situation.	More difficult to learn One must establish their own rules; rules are to be rediscovered. New situations are more easily handled since one knows which rules and procedures are feasible.
Rewards Rewards are more immediate, and more apparent since answers are reinforced immediately.	Rewards Rewards are less tangible, but at least equally effective. Satisfaction that one does know how to solve the problem, and why.

INSTRUMENTAL	RELATIONAL
Quickly forgotten If rules are unrelated to one another retention is low: learning is inefficient in the long run.	Greater retention Rules are related to one another, retention is greater and learning more efficient.
Once forgotten, a rule has to be learnt again and takes about the same time as the initial learning.	Once forgotten, a rule is relearned quickly and easily.
External motivation Teaching children rules has to rely on external motivations - the teacher's authority, marks, stars, fear.	Internal motivation Learning and understanding are goals in themselves. Satisfaction from one's efforts encourages further effort.

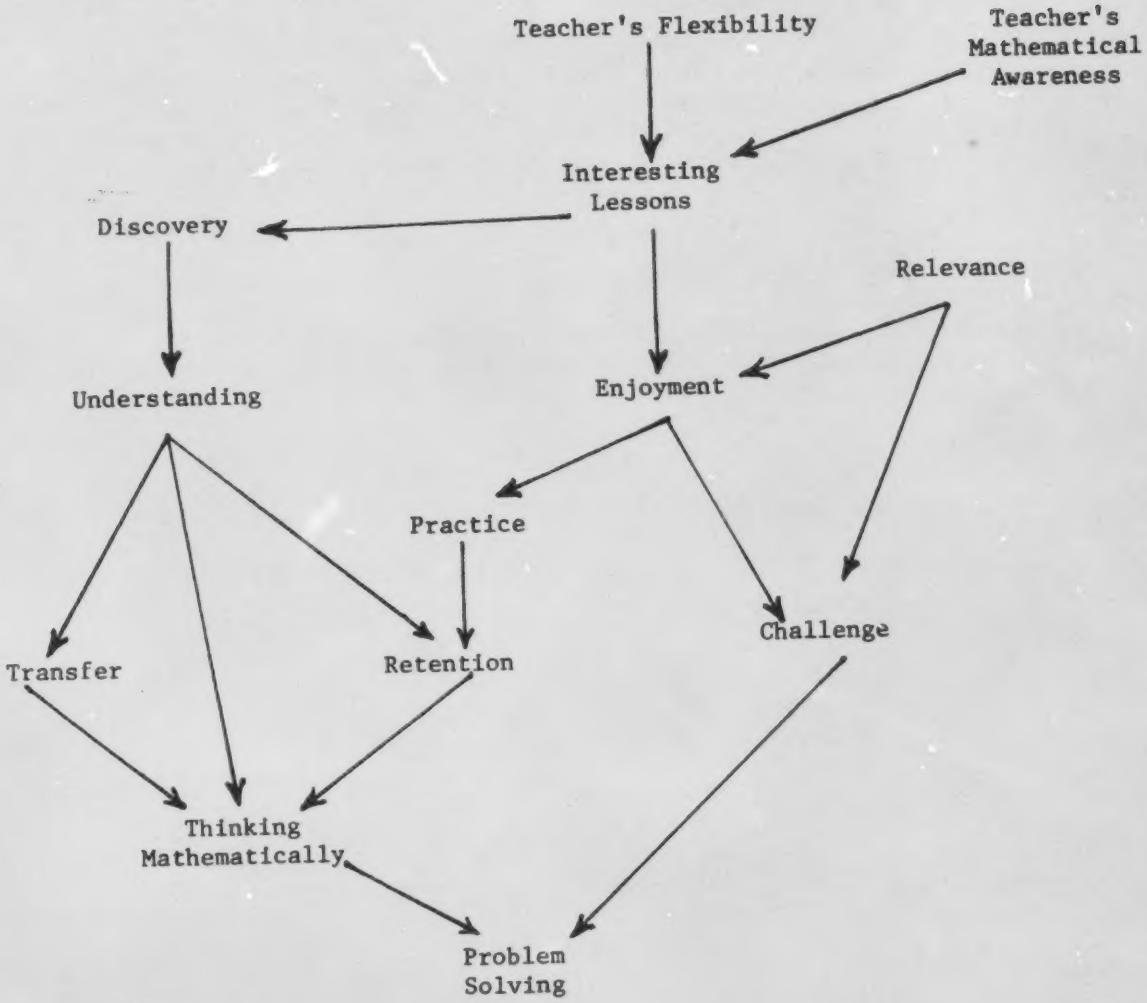
Which type of understanding teachers aim for is a reflection of many of the value judgements that were mentioned in the previous section on traditional and contemporary styles of teaching mathematics. In comparing this table with the table from the previous section, one can see that traditional teachers would favour instrumental understanding and contemporary teachers would emphasise relational understanding.

EFFECTIVE MATHEMATICS TEACHING

Each of the previous sections has been interpreted in terms of the dichotomies that can be seen to exist in the way mathematics teaching and learning is perceived to occur most efficiently. That is, the manner in which theorists believe children learn mathematics can loosely be described as either associationist or developmentalist; the manner in which mathematics is taught can be described as either traditional or contemporary; and the type of understandings about mathematics that teachers want children to have may be categorised as either instrumental or relational.

Each of these pairs of dichotomies can be viewed in another manner. It may be argued that a teacher who teaches mathematics in a traditional manner, has a predisposition, either intentional or unconscious, both to follow associationist principles of learning and to encourage instrumental understanding. Similarly, one may argue that the teacher more attuned to contemporary ideas in mathematics education will teach in a contemporary manner, and will be predisposed to follow developmental principles and to encourage relational understanding.

Figure 1



(Interpret "⟶" to mean "is likely to encourage")

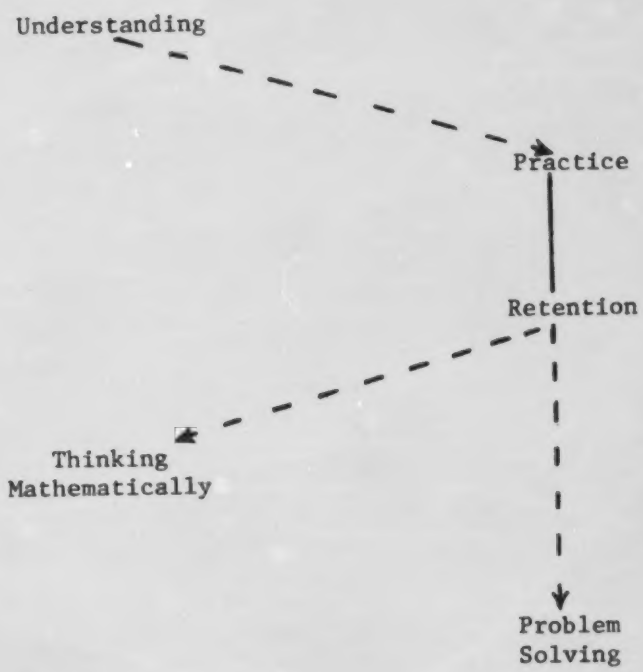
Figure 1 represents the inter-relatedness of many of the ideas important in the teaching and learning of mathematics. A teacher who is aware of the contemporary issues in mathematics education is likely to have a high mathematical awareness. This, together with the higher propensity towards flexibility typical of the more informed and commited teachers, increases the likelihood that mathematics lessons will be interesting to the learners and that these lessons will involve a guided discovery approach using concrete referrents: leading to both enjoyment and understanding on the parts of the learners. Such teachers will also be likely to seek out mathematics that is relevant to the learners.

Taken together, understanding, relevance and enjoyment are likely to encourage further learning in mathematics: learning that will be efficient and accompanied by an expectation of success. Understanding encourages retention of learned material, and raises the possibility of transferring learning to a new situation: and all of these encourage the learner to think mathematically. If a learner enjoys mathematics he is likely both to be prepared to practice skills, and so increase the likelihood of retention, and to accept the challenge involved in mathematical learning.

All these things taken together are likely to enable learners to solve novel problems, the pièce de résistance of mathematics learning. The possibility of all these things occurring is greater if the teacher follows a contemporary approach, is predisposed to developmental theories of learning and aims for relational understanding.

Contrast those views expressed above with the predispositions of a traditional teacher of mathematics who follows associationists theories and who emphasises instrumental understanding: such a situation is represented in Figure 2.

Figure 2



(Interpret ——→ to mean 'is likely to encourage')
(and - - - → to mean 'will possibly encourage')

Figure 2 asserts that traditional teachers are concerned almost solely with children obtaining correct answers through a series of practice questions aimed at helping children's retention. These teachers may hope that practice and retention lead to understanding, to learners thinking mathematically and to problem solving abilities: but the emphasis they give to this is less significant then those contemporary teachers described above.

Assertions: Mathematics teaching and learning does not take place in a vacuum. Each of these assertions listed below is relevant to effective mathematics teaching.

1. Each child is unique.

   Any group of children will be comprised of individuals who are at a variety of stages in their mathematical understandings, and who are developing at different rates.

2. No one method of teaching mathematics is superior to all others. There is no one right method of teaching mathematics.

   Contemporary approaches to the teaching of mathematics can be argued to be superior to traditional approaches, but the term contemporary approaches implies a whole range of teaching methods.

3. Effective mathematics teaching requires

   that teachers have a positive mental attitude towards the teaching of mathematics, towards the likelihood of their pupils learning mathematics successfully, and in their general behaviour towards their pupils;

   accepting one's right as a teacher to have authority over those one teaches, and requires teachers to use this authority humanely and effectively.

4. Teachers have multiple roles to play simultaneously: they are advisers, friends, directors, consultants and decision makers.

DIAGNOSTIC AND PRESCRIPTIVE MATHEMATICS

Significant numbers of teachers believe that a child's ability to obtain a correct answer indicates that the child has understood this mathematical topic. A view that has long persisted is that if a child needs remediation in mathematics his problems will be overcome by practice in those questions he cannot do. This procedure encourages rote memorisation which is of little benefit to the child in the long run, particularly since subsequent mathematical material proves too difficult given this weak foundation.

Today's teacher has a basic responsibility to provide the physical environment and a questioning technique that provokes the logical development of processes necessary for conceptual development. In order to determine which procedures are meaningful at a particular time the teacher must carry out some form of diagnosis.

Diagnosis is the procedure involved in obtaining essential data and background information about each learner, and the process of analyzing that data and information. It is essential that this operation is carried out, as the only way a teacher can know about her own pupils and plan her programme of instruction, is to assess their performance herself, or have them assessed by those specifically trained in the assessment and diagnosis of learning difficulties, e.g. school counsellor.

Many of the difficulties which some children experience in the learning of mathematics may be avoided if the teacher

- determines the level of learning at which the child is performing;
- analyse the task to be learned so that the learning of the concept may be developed in short steps based on what the child already knows;
- determines the prerequisites which the child must know before proceeding to a more complex concept;

- varies the approaches and activities to suit the learning styles of the children;
- encourages and listens to children, and creates a successful learning environment;
- puts mathematics into a realistic setting that provides relevance to the everyday world;
- realises that the child may have a fear of mathematics derived from experiences in a previous grade or from the attitude conveyed by teachers or parents.

The basic assumption for helping students overcome a fear of mathematics is that a personal relationship of trust must be established between the teacher and student. Once this relationship has been established it is usual to begin instruction utilizing informal methods. As students learn from spontaneous experiences, popular games, hobbies and related activities can then provide a basis for instruction. The programme must provide for initial success to reassure the pupil and should be centred on an approach designed to maintain a high interest level.

FAILURE IN MATHEMATICS

Those factors leading to failure in mathematics may be categorised as follows ...

1. Failure centred on the learner, but not mathematics specific:

   memory capacity
   problem solving ability
   language
   behaviour patterns
   attitudes and interests
   confidence
   motivation

2. Failure brought about by environmental factors:

   physical needs
   emotional needs
   socio-economic variables

3. Failure resulting from instructional weaknesses:

   effective classroom management techniques;
   taking into account learners' individual differences;
   alternative instructional approaches - providing verbal concrete, pictorial and symbolic representations.

4. Failure specific to the learner's inability to grasp mathematics:

   confusions, partial learning, or misunderstandings of concepts presently being studied;
   difficulties resulting from gaps in the learner's understandings .... where these gaps are few;
   difficulties resulting from missing prerequisites, where these gaps in the learner's understandings are many.

DIAGNOSTIC AND PRESCRIPTIVE APPROACHES

1. To have teachers do better what they already do well.

2. To identify students' (and teachers') strengths as well as weaknesses. For example, are the learner's problems

   (a) that the general level of achievement in mathematics lags behind achievement in other subjects;

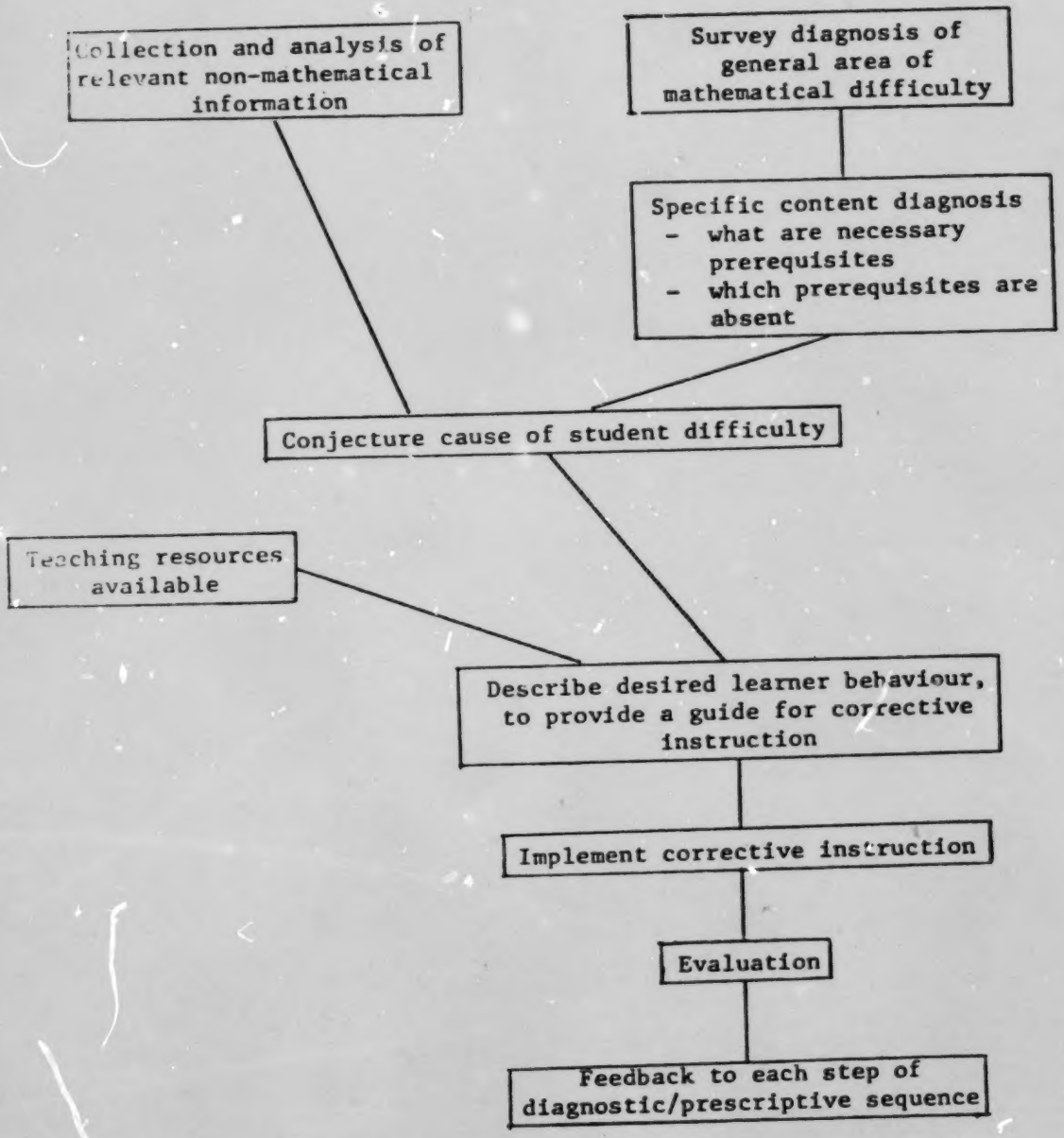
   (b) related only to specific topics in mathematics;

   (c) the result of mathematics anxiety: where the student functions satisfactorily quantitatively in normal situations, but conscious association with mathematics causes anxiety.

3. To have students misunderstanding corrected, not to simply achieve the correct answer but to emphasise the answer to the question "why did this error occur?".

4. To reteach successfully what was not well taught or well learned during the initial teaching.

A DIAGNOSTIC/PRESCRIPTIVE SEQUENCE


Collection and analysis of relevant non-mathematical information
Survey diagnosis of general area of mathematical difficulty
Specific content diagnosis
- what are necessary prerequisites
- which prerequisites are absent
Conjecture cause of student difficulty
Teaching resources available
Describe desired learner behaviour, to provide a guide for corrective instruction
Implement corrective instruction
Evaluation
Feedback to each step of diagnostic/prescriptive sequence

CASE STUDIES

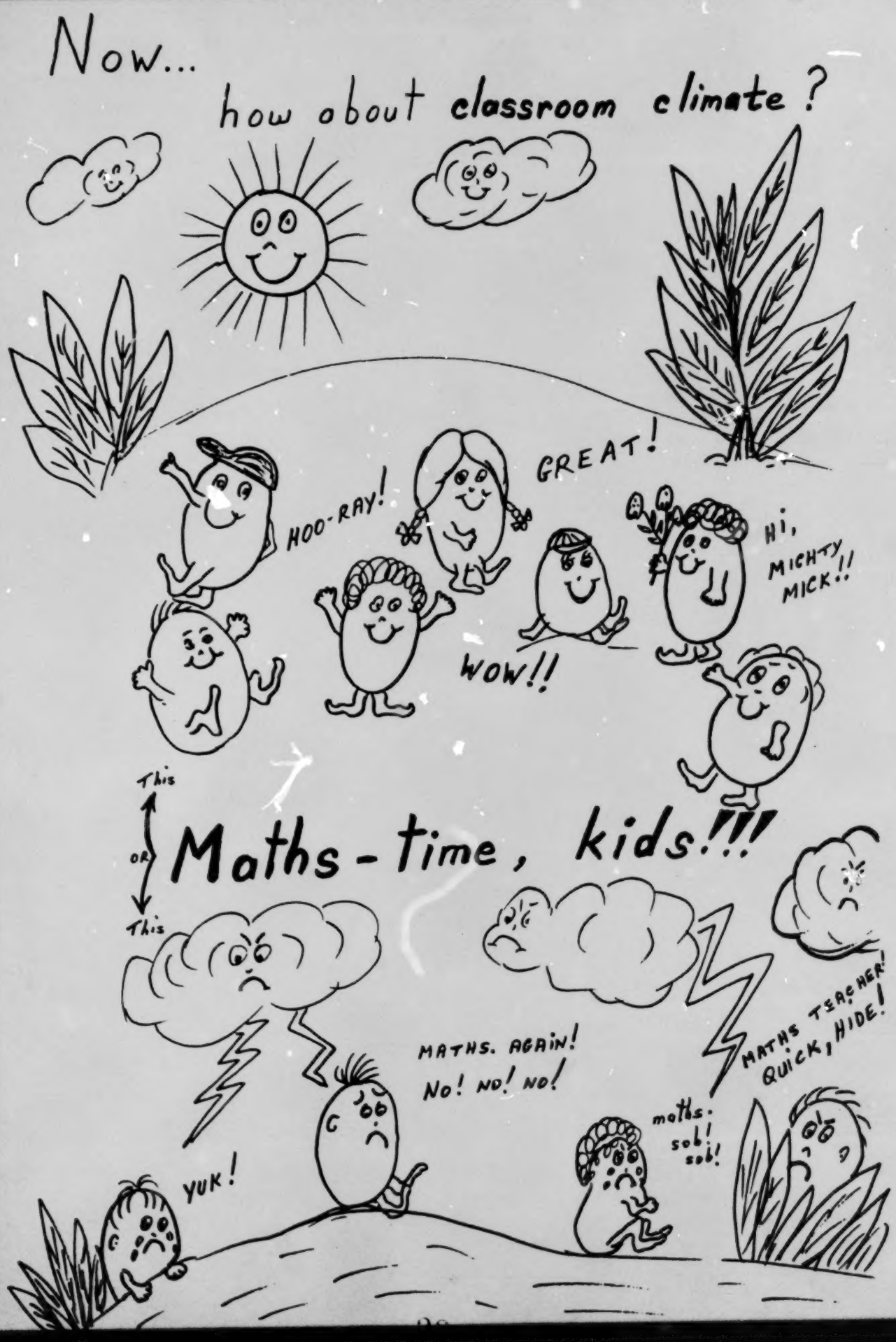
J. aged ten, was having difficulties in nearly all her mathematics. J. was above average in general intelligence but immediately froze and became disinterested if mathematics was mentioned. After six half hour individual sessions in mathematics J. was able to significantly improve her mathematics achievement: and her teacher reported that J. was more willing to try in mathematics even after making mistakes.

This diagnostic and prescriptive sequence gave J. the opportunity to look again at a range of very basic mathematical concepts that she had not understood when taught initially.

By the time ten year old S. had finished the tenth of twenty questions it was evident that his mathematical understandings had numerous gaps. S's teacher was trying to teach him the division algorithm, but with little success. An awareness of the hierarchical structure of the prerequisites to successfully performing a division algorism, together with a diagnosis of S's ability in these prerequisite skills, soon led to the underlying problem: a confusion in addition and subtraction operations.

Eleven year old C. was very self-conscious and embarrassed at being singled out for extra help in mathematics, and particularly at his inability to understand division. The program prescribed as a result of a diagnostic test initially emphasised informality and mathematical content with which he had previously experienced success. As he experienced success, he became more confident and more successful.

TEACHER ATTITUDE
AND
CLASSROOM CLIMATE


Now...
how about classroom climate?
HOO-RAY!
GREAT!
Hi, MICHTY MICK!!
WOW!!
This
OR
This
Maths-time, kids!!!
MATHS. AGAIN! NO! NO! NO!
MATHS TEACHER! QUICK, HIDE!
YUK!
maths. sob! sob!

What causes the climate?

Is it...? or is it...?

An AUTHORITARIAN MODEL of Teaching?

A PROBLEM-SOLVING MODEL?

Pupil involvement in discussions with teacher.
pupil-to-pupil discussions, helping each other.

Kids feeling free to ask questions

Teacher guidance of discussions and activities.

MIGHTY MICK, MATHS-MASTER and marvel!!

MATHS should be FUN!

MYOPIC MYRA

FUN, rubbish!!
PEACE and QUIET, please!
EVERY ONE working alone.
NO INTERRUPTIONS!
Silence! No talking.
Teacher dominance of class discussions.
Democracy is for governments, NOT Classrooms!!

(Teachers can be as short-sighted as Myra about what happens in their class rooms.)

We're the ones in the MIDDLE!!

SLOW-LEARNER SAM

HOO, BOY!!!

So .... who controls classroom climate??

YOU DO!

YOUR attitude to MATHS affects your pupils' attitudes to MATHS.!!

You were probably taught by MYRA

SO

Mick's attitude is new and frightens you.

BUT

Just because you didn't like Maths, why should the kids suffer?
If you make an effort and enjoy the lesson, the kids will, too!!

→ You CAN teach Maths with understanding —and enjoy it!— and CHILDREN can learn to understand and enjoy it, too!!

USE → inexpensive materials— and new techniques that are easy to find and use in lessons.

TRY → AN IN-SERVICE COURSE (you might enjoy it!)

↓

TEACHER PREPARATION need not be a drag!! use the KIDS! they love it: Charts; models; junk from home, magazines. Use the PLAYGROUND

IF...

Myra's Maths left you COLD, why not give

SOME IMPORTANT PRINCIPLES

MOTIVATION

ACTIVE LEARNING

COMMUNICATION

DISCUSSION and QUESTIONING

CLASSROOM ORGANISATION

THE BASICS

STAGES of DEVELOPMENT

EVALUATION — child and teacher

see over leaf for EXCITING details.....

Motivation

Of course I like to motivate!!

☆ ✓ (occasionally)

young children will try hard for these symbols of success.

Older children work hard to pass exams; to prepare for the future.

Sure! BUT how about SELF-MOTIVATION?

IF a problem comes from within a child's experience he can be self-motivated into wanting to solve it. The joy of learning through experience!!

Hurrumph! That's easy in theory! How do I do it?

Look around you, women! USE

the classroom, playground, home, sports carnivals, things that happen each day.

USE ACTIVE LEARNING!!!

BUT....

What is "Active Learning"?

Most children in primary school (and sometimes early high school) are at the concrete operational stage of development. These children learn through activity and involvement with real situations and real materials. DISCUSSION plays an important part in the development of thinking.

But... but there are no books to tell me what experiences and activities to use.... and DISCUSSIONS are so noisy!!!

Use the children's environment!
- people at home and school.
- places he knows well and can visit.
- things he can feel, hear, see, touch.

INTEGRATE with art, P.E., social science, EVERYTHING!!

How?
How?
How?

measuring — estimate... hands, feet, heads, height... then GRAPH the results.

USE

the TELE.
- SURVEY viewing habits between 4pm - 6p.m.

OR

- work out the time allocated to commercials, current affairs, kid's shows.

the SUPERMARKET.
Go shopping!! No time? Then use brochures and pretend. Let kids make up cheques and spend $50 on groceries. stacks of +, −, × and ÷ there!!

Mmmm....
SOUNDS interesting —
but how do I REALLY
organise ACTIVE LEARNING?

EASY!!

Let's take Fractions...

FIRST...

ACTIVITY and EXPERIENCE

Share pencils among, say, six children.
Divide the class into 6 groups.
Cut up a circle into 6 parts (pretend it's a CAKE)
Play with the parts... one part of 6
3 parts of 6...etc.

NEXT... DISCOVERY

Teacher-directed by use of certain arrangements of data or open-ended questions.

e.g. what happens if we have to share our cake among 2 children, 4 children, 6 children?

THEN... ACQUISITION of facts and knowledge.

What have we found out?

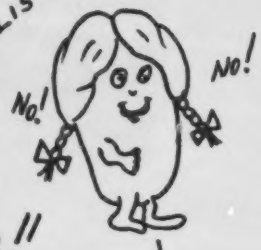
$\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$; $\frac{2}{6} = \frac{1}{3}$; $\frac{3}{6} = \frac{1}{2}$; $\frac{6}{6} = 1$.

Hey! Maths is meaningful!
Maths is fun!
I'm starting to
UNDERSTAND IT!!!

COMMUNICATION

Don't interrupt.!!

The trouble with you, SUZIE, is you DON'T LISTEN!!



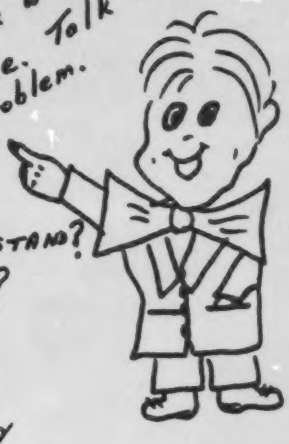
TRY HARDER!!
No talking!
Can you do it? →

NO!

← understand?

ANY QUESTIONS?

Use these counters, Suzie. Work with SAM, if you like. Talk about the problem.



UNDERSTAND? NO?

Right! Let's try M.A.B blocks

Okay. We'll try Cuisenaire rods or PLAY MONEY or DICE GAMES or NEWS-PAPERS or ABACI or MAGAZINES.

Of course you can! I showed you last week. LISTEN!

what's this for? →

Put your hand down! Work quietly.

LISTENING POSTS! UNDERSTAND, NOW SUZIE?

YES!! thank you!

DO you UNDERSTAND?

What is questioning?

A question is a question!

what are seven eights?

ANSWER. 56.

CLOSED QUESTIONS.

BUT ... how about OPEN-ENDED questions?

What happens if...?
But why...?
What do you think?

leads to

INVESTIGATION

AND the children's questions?

INVALUABLE! They arise from various individual needs e.g. experience, curiosity, emotional need for attention.
ANSWERING them gives a significant contribution to the child's eventual stability.

And, discussion?

Ensure that there is something worthwhile to discuss. This leads to social, emotional and intellectual growth.

Create an atmosphere where everyone has something to offer and is entitled to hold an opinion.

What types of discussion are there?

Teacher – child
Teacher – group
Child – child.

CLASSROOM ORGANISATION

How do I organise my class to do all that?

The range of abilities is so great!!

I like them to open their texts and work through problems 1 to 10 — quietly!

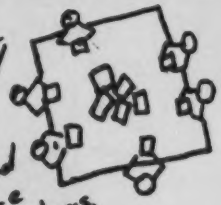
EASY!

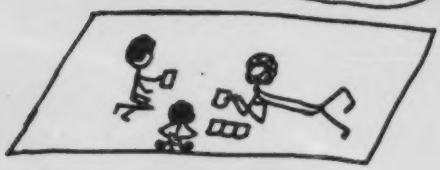
ORGANISE GROUPS!

ALSO — make maths equipment accessible!

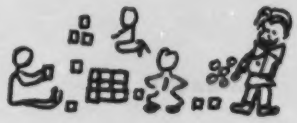
use — work sheets.
puzzle sheets.
games.

One group is working on activity cards. They understand subtraction and are doing more advanced problems.

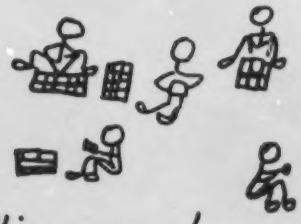




This group is playing a game. They don't know their subtraction facts and need reinforcement.



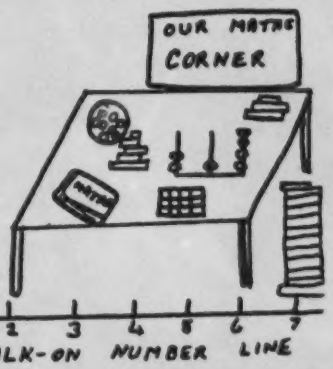
One group is using DIENES BLOCKS, Rods + COUNTERS. They do not understand subtraction and The TEACHER is helping them.



This group have caught on but need reinforcement. They are using concrete materials.

MATHS EQUIPMENT

OR

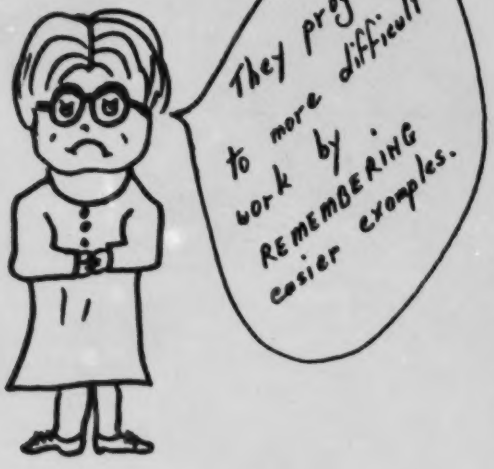


Be a good scout...
Be prepared for a maths lesson!

BASICS

Good MATHS students.....

Remember...
tables
theorems
formulae...
they get to understand them—
EVENTUALLY!!



RUBBISH, Mick!
They learn best by repetition and using their memories!

Repeat after me!
Say tables..
again and
again and
again!
That's effective learning!

NO, THEY DON'T, MYRA!
Children must understand exactly what they do, AS THEY DO IT!!! —
seriation,
matching, pairing,
sets, attributes,
number lines,
classifications,
number facts,
addition,
subtraction,
multiplication,
division.

Remembering TABLES, too.
BUT only if they understand WHY first.!!
MEMORY is fine, but it doesn't help in many PROBLEM-SOLVING situations.

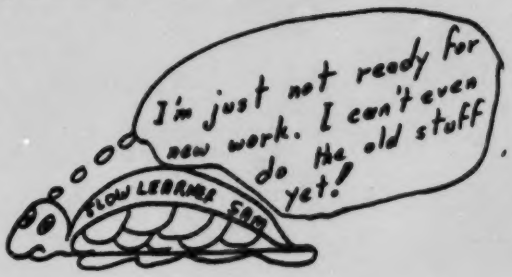
No! no! no!
They must UNDERSTAND
Rote learning is for parrots!
ENCOUNTER
UNDERSTANDING
REINFORCEMENT
That's what they need!
Remember, Myra.
MATHS is FUN!!

Now for

STAGES of DEVELOPMENT

Piaget... who's he again?

I press on regardless! They catch up as best they can. With all the other children and my syllabus commitments, I don't have time to waste on one or two slow ones!



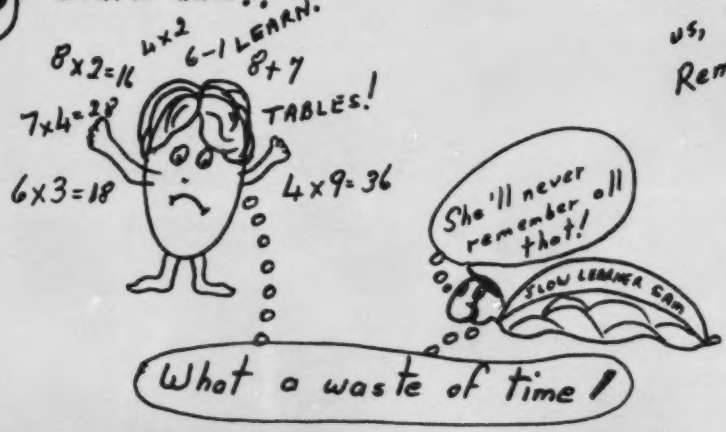
Poor SAM! Poor kids! It's like beating your head against a brick wall, trying to teach a concept to a child who is not at that stage of understanding.

It sinks in eventually. Besides, the teacher they have next year can often correct any problems they have this year.

That's a cop-out attitude! It results in a frustrated child who may not even TRY next year.

ALRIGHT! What are these STAGES of DEVELOPMENT?

LEARN! STUPID GIRL!!!



PIAGET! He's done all the work for us, even the tests. Remember... pre-operational stage? Concrete operations? Formal operations?!

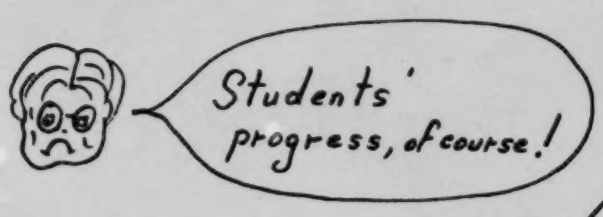
Bone-up on them!!

DID IT WORK?

Are you ready for the awful word?

EVALUATION

What do we Evaluate?



To make sure we're doing our job!

Try video-taping one of your lessons (if you're game!)

Child and teacher evaluation sheets like the one on the next page (again, if you're game!)

If you're really game, invite another teacher or the principal to watch.

Content Evaluation.

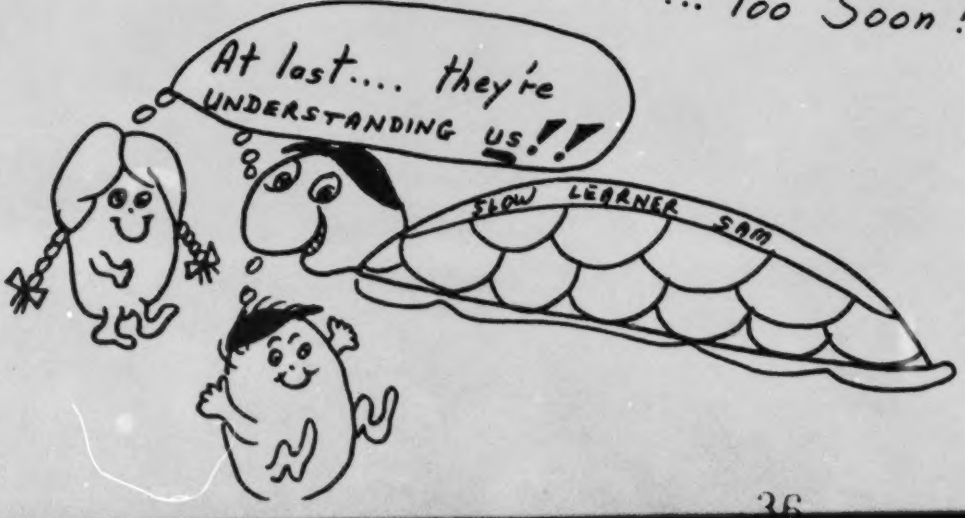
I stick to the SYLLABUS — EXACTLY!!
You may not get all the way through it. Remember those STAGES of DEVELOPMENT!
? ? ?
You mean... IGNORE the syllabus?

OF COURSE NOT!!!
After all, it does have all the operations in their proper sequence.

REMEMBER

You might be trying to teach

Too Much... Too Soon!!



CASE STUDIES

R. aged ten, was quite below average in mathematics and had a negative attitude towards the subject. Enquiries revealed that her classmates generally agreed with her feelings about mathematics: and, without exception, placed the blame on their teacher in their previous class. The teacher taught little mathematics, the children soon learned that mathematics was something to be avoided.

Seven year old D. is described by his teacher as lazy, inattentive and uncooperative. The teacher seems happy with his formal approach to lessons, and continues to discourage discussion and stifles children's spontaneous reactions to his lessons.

D. has little foundation upon which to build further skills and knowledge: he is frustrated and bored by school. Consequently he does not try hard and is convinced he cannot learn. During his brief remediation program D. readily responded to praise and other reinforcement. For D. to learn efficiently requires a warmer, more spontaneous climate where he is able to express his views in a receptive atmosphere

ATTITUDES AND MATHEMATICS

In the last decade or two educators have become more and more concerned with the affective outcomes of educational programs. The development of a positive attitude toward the subject being studied is probably one of the most prevalent educational goals because many teachers believe that a student's attitudes toward a school subject will affect that student's achievement in the subject.

Although all teachers are interested in pupils' attitudes toward the subject they are teaching, teachers of mathematics are particularly concerned about pupils' feelings about their subject because mathematics has a reputation for being unpopular. Most of us know someone who "hates maths" or "freezes up" when faced with a simple arithmetic problem.

Therefore, the affective objective of the teaching of mathematics must be that the student will want to spend time doing mathematics and enjoy doing mathematics.

To achieve this objective we must look at work done by researchers on attitudes toward mathematics. From their studies we should attempt to answer such questions as why attitudes are measured, how they are measured, how reliable are the measuring instruments, when are attitudes formed, and how are they formed. Maybe then we can better understand attitudes, and try to show students how enjoyable mathematics can be, and how useful.

WHY ARE ATTITUDES MEASURED?

Interest in the students' attitudes toward mathematics has occasioned numerous studies. Some suggested reasons for this interest are:

(1) the desire of teachers to find out how their students feel about mathematics and the effects the mathematics courses they are teaching are having on students' attitudes toward mathematics.

(2) to use the students' attitude results as predictors of achievement.

(3) to test a hypothesis that a poor attitude to mathematics might actually be damaging to the personality, which if proved true, would require urgent remediation.

HOW ARE ATTITUDES MEASURED?

In attempting to measure attitudes many approaches have been tried including behavioural observations, interviews, questionnaires, rank ordering of school subjects, attitude scales, sentence completions, picture preferences and content analysis of essays.

The two most common methods for gathering data are self-report techniques and observational rating techniques.

Self-report techniques usually involve paper-and-pencil instruments that require students to report how they feel about a subject. Several kinds of items are used on such instruments:

(a) Open-form items, e.g. "What do you most enjoy about mathematics classes?"

(b) Checklist items, e.g., "Do you think mathematics is fun?"

(c) Likert-scale items, e.g., "Word problems show me how useful mathematics can be".

Strongly Agree _____ Agree _____ Undecided _____ Disagree _____ Strongly Disagree _____

(d) Semantic differential items, e.g.,

Arithmetic

good:_____:_____:_____:_____:_____:bad

(The student ticks the part of the line that shows his "feelings" about arithmetic)

Observational rating techniques usually require the teacher or some other person to observe the students and record their behaviours. Different types of observations can be made:

(a) How many homework assignments a student finishes

(b) How often a student disrupts the class

(c) Whether a student uses "free" time in mathematics classes to do extra maths work or something else.

HOW RELIABLE ARE ATTITUDE TESTS?

Both of the above techniques have advantages and disadvantages. Results of self-report type tests show that the reliability and validity of these tests vary somewhat with grade level, being generally more reliable and valid in high school and college. The reasons for this are that not only do attitudes become more stable with maturity, but the degree of self-insight and conscientiousness with which students can express their attitudes increases with age. In addition, problems of readability and interpretability of self-report inventories are more serious in the lower grades.

However, there are three problems with the observational appraoch that often make self-report techniques preferable. They are:

(1) identifying behaviours that reflect a student's attitude toward mathematics is difficult, e.g., just because a child wants to go outside and play instead of doing maths homework doesn't necessarily mean he doesn't like maths.

(2) standardizing your observation is difficult, e.g. comparing one pupil at 9am with one at 3.15pm.

(3) deciding how to quantify the behaviour is difficult, e.g., how do you measure grimaces?

The major difficulty with all attitude tests, especially those in the primary school, is that it is difficult to know where the real feelings are being expressed and whether the student is saying what he thinks he is expected to say—what adults will reward.

WHEN ARE ATTITUDES FORMED?

Some studies say that children are already forming opinions about mathematics by third grade, and very soon after that they develop quite strong opinions about mathematics either for or against, and that it is difficult for the teacher to change their attitudes.

Other studies indicate that children's attitudes don't appear to be as stable in the primary grades as they are in the secondary school, which would seem to indicate that it is possible for the teacher to at least modify attitudes to mathematics in the primary school.

Therefore, the early years of the child's education are crucial in the formation of attitudes in the basic fields of mathematics and language.

HOW ARE ATTITUDES FORMED?

The formation of attitude in individuals must be influenced by many factors such as IQ, personality, home background and environment. In trying to explain why there are apparent differences in the average mathematical ability and attitude scores of boys and girls, various views have been put forward: including genetic, psychological, and cultural based hypotheses. The genetic hypothesis sees numerical ability as a sex-linked recessive gene. Of the psychological theories one puts the blame for poor mathematics ability in some boys on the absence of father in the family situation. Cultural conditioning is put forward as the reason why boys exhibit superior attitudes and achievement in mathematics in high school whereas in primary schools differences between boys and girls in this area are not noticeable.

It is probably unlikely that attitude towards mathematics can be attributed to a single cause. Attitude towards mathematics is probably a cumulative phenomenon caused by one experience building upon another; individual attitudes, shaped by personality and background, are brought to school where they can be reinforced or modified.

It would be expected that parents' attitudes to mathematics would have some effect on children's developing attitudes and this is confirmed to some extent by studies. Mothers' attitudes towards mathematics effected the attitude of their sons and daughters, whereas fathers' attitudes towards mathematics seem to have more influence on sons' achievements in mathematics. Another study showed that for early high school students, mothers' interest and attitude was more significant for females and fathers' attitude more significant for males.

Studies have also sought to find relationships between attitude to mathematics and other personal characteristics. Aiken found, for example, that people who have a positive attitude to mathematics like detailed work and problem solving. A positive attitude to mathematics tends to be inversely related to interest in language arts, social studies and other verbal pursuits. This would certainly be true of some students but many in the top level of the class are found to be equally good in the fields of mathematics and language. One student, now fourteen, fits the category of a student who excels in language but who exhibits a poor attitude to mathematics. He finds great difficulty with problem solving and only excels in mathematics in areas where rote learning is an advantage, e.g., tables and formulae. He has always dreaded the mathematics lesson but is happier now that he has been put into a lower level mathematics class. For him mathematics has always been baffling and frustrating, yet he enjoys the tactics and strategies of war games and is quite happy working various calculations to arrive at particular military strengths.

Within the school environment, the teacher emerges as the most significant influence on student attitudes to mathematics. It seems that if attitude and teaching methods show that the teacher is interested and confident as well as understanding, the pupils are more likely to develop positive attitudes to mathematics. An example of this is a young woman who has a positive attitude to mathematics and, when asked to describe her mathematics teachers at school, says they were kind, helpful and knew what they were talking about.

Within the mathematics programme many different techniques can be tried in an effort to improve children's attitudes toward arithmetic and mathematics. These include counselling, mathematics games, special courses, etc. It was hoped that modern mathematics programmes would be an effective means of improving attitudes, but results have been disappointing.

ARE ATTITUDE AND ACHIEVEMENT RELATED?

Commonsense would make many people assume that a student who has a good attitude toward mathematics would also do well in the subject. This assumption has been tested by a number of studies where researchers have tried to measure the relationship between attitude and achievement. Even where a relationship has been found to exist the question remains - does attitude affect achievement or does attitude follow achievement?

Studies at the secondary school level have shown a significant link between attitude and achievement in mathematics. In general, it is difficult to establish reliable relationships between attitude and achievement in mathematics during the primary school and also to predict what attitudes will be in the secondary stage of education.

Some researchers concluded that ability is usually a more important predictor of achievement than is attitude, with only extreme attitudes really having an effect on achievement. Scores on attitude scales may be better predictors of whether students will want to do mathematics and continue to do mathematics at a later stage.

At present, research has not provided a basis for concluding that the attitudes toward mathematics and achievement in mathematics are causally related.

SIGNIFICANCE OF FINDINGS FOR THE CLASSROOM TEACHER?

Teachers must be aware that there are numerous variables affecting the formation of attitudes. They must realise that children can be turned off mathematics before they reach a more interesting stage in the subject. Also that negative attitudes will develop if the children:

(a) have to learn a lot of dull facts,

(b) don't understand what they are doing,

(c) feel baffled and frustrated,

(d) don't see any relevance between mathematics and real life situation, and

(e) are afraid of failure.

Therefore, teachers should provide experiences that are rewarding and challenging, and ensure that the mathematics experience is seen as pleasant, interesting and of potential value to the child. The child who receives positive reinforcement about himself as a mathematics student will be more likely to persevere and to pursue mathematical activities throughout his school career. Thus we will have achieved our objective.

Attitudes, positive or negative, have only a slight causal influence on how much mathematics is learned, remembered and used, but we could safely assume that a negative attitude to mathematics would affect the child's willingness to cooperate in the learning process and to persevere in mathematics by choice. As it seems that positive or negative attitudes to mathematics can be established at quite a young age, it is vital that the primary school develops and continually evaluates a programme that provides satisfactory mathematics experiences for all levels of natural ability.

As we have noted, it is the teacher who is the crucial factor in the development of attitudes and so it is the teacher who can try to ascertain the attitudes the child brings to school and try to develop in the child a positive self-concept with regard to mathematics.

Attitude tests can be administered by the teacher as long as the teacher is aware of the disadvantages of such tests, and provided the tests are designed to suit the reading level of the pupils they are written for. Then teachers may be able to organise their mathematics programmes with some regard to their pupils' attitudes, or interests.

CASE STUDIES

S. aged ten, reported that he didn't like mathematics. A simple attitude scale asking questions about a variety of mathematics topics was administered. The results revealed S's dislike of mathematics was really a disliking of only division: a brief remediation program in this skill soon overcame his difficulties and led to a more positive attitude towards mathematics as a whole.

The same youngster had an interesting attitude towards the use of concrete materials.

In the remediation lesson S. was quite happy to use a large plastic number line and MAB equipment to help his learning. On one occasion the MAB units were missing, as soon as S. saw the teacher reaching for cuisenaire rods he reacted with immediate disgust: "they are for little kids". Immediately after this though, S. was happily using lego building blocks to help in his calculations.

A student reported the case of a friend she was visiting. The two adults were looking through the friend's daughter's mathematics exercise book. The friend showed considerable disgust with one particular division algorithm that had taken almost a page to complete: her comment was "I was shown a better and shorter way at school". She seemed little concerned with her daughter's understanding, and not at all aware that there may be more than one way to arrive at an answer correctly.

LANGUAGE AND MATHEMATICS

Many people believe language and mathematics to be two separate areas of study, that is language dealing with words, and mathematics with numbers. However, mathematics has a language all of its own; a second language entailing the introduction of new symbols as well as expanding and giving new connotations to old symbols. Areas covered in this paper will include

- Language Development
- Language in Mathematics
- Problems Arising from Language in Mathematics -
  Fear of Mathematics; The Inability to Read; The Inability to Comprehend Problems; Variations in the Meanings of Words in the Mathematics Context; Syntax in Mathematics Texts
- Remediation of Language Associated Problems -
  Fear of Mathematics; The Inability to Read; The Inability to Comprehend Problems; Variations in the Meanings of Words in the Mathematics Context; Syntax in Mathematics Texts
- Recording
- The Appearance of Text

LANGUAGE DEVELOPMENT

From the cradle, parents assume the responsibility of teaching their child their natural language. When they come to school, this is further developed and is reinforced by their success, their confidence, and the sound understanding that the child experiences. This is clearly seen in the care and attention given to the construction and development of a reading scheme throughout a school.

"Usually a good scheme pays particular attention to a number of factors which are worth noting:

(1) Initially the words used are common to the everyday needs of the child; complex and lengthy words are avoided; words that are abstruse or have a double meaning are left out;

(2) Sentences are carefully constructed; usually simple and limited in length; clear in meaning and often supported by illustration;

(3) The content of the scheme is relevant to the child's experiences; the ideas expressed relate to their understanding the interest level is usually high and the material appealing to the reader;

(4) The typographical presentation is bold; letter characters are easily identified and the general appearance of a page is perceptively pleasing".

All these factors are geared to equip the child with a sound basis on which to expand their growing needs in language.

LANGUAGE IN MATHEMATICS

Mathematical language often seems foreign to children, and they may fear it, because much of it is not within their range of everyday experience. Therefore, it is the teacher's duty to expose the child to a variety of experiences so that he becomes conversant with the language involved, and gradually acquires meaning in the mathematical sense.

Because mathematics is fundamentally an artificial symbolic language, greater care should be given to the structure and presentation of mathematical material. For example you would not expect a beginner to be able to read a book with small print and no illustrations, yet we may expect a beginner in maths to perform a task which is equally difficult. Therefore a sight vocabulary of words and phrases which are to be read and comprehended quickly may be a way of avoiding unrealistic expectations. An agreed vocabulary and related symbolism needs to be developed for each major topic within the mathematical experiences of the child.

Many teachers fail to realize that young children have a limited and functional vocabulary and are unable to cope with logical alternatives as a matter of choice. The table below shows eighteen different ways addition was presented within eight popular primary mathematics texts. One text, designed for the average and less than average child, used fourteen of the eighteen alternatives in two pages.

3, 5
5 and 3 equal
5 + 3
5 plus 3
3 greater than 5
The total of 5 and 3 is

5 add 3
Add 3 and 5
The sum of 5 and 3 is

5 and 3 together make
5 and 3
5 more than 3
Add 5 to 3
5 count on 3
Write the number facts for 8 using 5 and 3
What must be added to 5 to make 8
Increase 5 by 3
How many are there altogether?
(Picture of 5 and 3 items)

Imagine the affect this could have on a young child. As well as having to cope with this type of situation, the child also has to cope with transforming their natural language into an abstract precise vocabulary. They are also using symbols that they have seen before and giving them mathematical meaning: for example '+' could mean ambulance in their everyday experience, but when surrounded by numbers in a mathematical context, its meaning changes.

PROBLEMS ARISING FROM LANGUAGE IN MATHEMATICS

Fear of Mathematics

The child may fear mathematics and symbolism because it is a totally new way of viewing his environment, and in order to achieve at school, he must learn to master it. To some children, this is a seemingly insurmountable problem because they have been left behind before they even start. This is probably due to the fact that they have not yet mastered their natural language sufficiently enough to cope with this new mathematical language.

> T. will respond to a sentence written on the board that contains her name, but as soon as a symbol such as "=" or "+" is written, she immediately becomes uncertain and assumes a defeatist attitude.

The Inability to Read

Many children have difficulty with mathematics simply because they cannot read the instructions given.

- J. recalls: "He did the reading and I did the maths, so we've done it".
- S. could not read the problem; but when it was verbalized for her, she could do it without difficulty.

Other children may be able to read natural language but fail to read mathematical language, because the same technique doesn't apply. For example, in natural language you read from left to right, but in mathematical language disturbing patterns are introduced

$4 + 7 =$ left to right

but

$\begin{array}{r} 4 \\ \underline{7} \; + \end{array}$ top to bottom
or bottom to top

Another problem is that in the reading of natural language, the child can guess what is coming up by what has gone before, but mathematics has its own reading patterns.

The Inability to Comprehend Problems

Problems can arise if the teacher, knowing that the child is able to read, presents him with a mathematical problem and then is confronted with the fact that the child does not comprehend what he is reading.

The child may not have a sound basis in deciphering mathematical terms. Children need to understand the words and terms frequently used in mathematics without hesitation and without excessive strain.

We must not forget a child's meaning for words develop over a period of time. Use of a word by a child cannot be accepted as indicating that the child possesses a full meaning for the word. Accurate use does not guarantee understanding.

> Charles, when counting five marbles, four red ones and one blue one, counted the blue one last so that it could be called "five". For a time afterwards, Charles called any blue marble "five".

Teachers must be aware that children's comprehension of mathematical problems must be developed in sequential steps rather than working from the assumption that, just because the child can say the problem and read it, he has all the hidden meanings of the problem at his disposal.

Variations in the Meanings of Words in the Mathematics Context

Keeping in mind that variation of text and differing methodologies are crucial to the interest factor in mathematics, which in turn is vital to a successful text, we, as teachers, must identify whether this variation in the text is exposing the child to a single concept or whether he is being over exposed to too many meanings of that concept in a short space.

Oblong and rectangle are words used to refer to the same sort of shape. Unless a person clearly understands the meaning of one word first, it is difficult to define the second word simply by using the first. When one

of the words has two meanings the situation can become even more confusing for the student: for example, is an upright citizen a vertical one?

Syntax in Mathematical Texts

Teachers should be aware that in applying a natural language sentence to a mathematical sentence we are trying to force one form of syntax onto another. Problems will most likely arise because mathematics has a syntax all of its own and we find that there are several verbal intepretations - all of which are correct - which can be applied to one mathematical statement. The translation of the statement will depend on the person who is reading it. For example, consider the ways we talk about "6 - 2 = 4"

- Six take away/minus two is equal to four.
- Two taken away from six is equal to four.
- The difference between six and two is four.
- Six and four, under the operation of subtraction, is equal to two.

REMEDIATION OF LANGUAGE ASSOCIATED PROBLEMS

Fear of Mathematics

When confronted with a child who fears mathematics, the teacher should try to overcome this fear by building up a good rapport with him. By giving him examples of work in which he is sure to succeed, he will regain confidence in himself, and from there, may build up a sound understanding of the subject.

The Inability to Read

This inability to read is not confined to maths, considerable attention must be given to rectify this problem in order that the child be able to expand his knowledge in all subjects, including maths.

While the child is at the pre-reading stage, his mathematics ability may still be extended by verbalizing problems, minimizing written instructions and keeping them within his everyday experiences.

The children should be encouraged to talk about maths notions before they are expected to read them, and to analyze and discuss maths ideas. They should also describe ideas and give explanations in their own words. Verbs in mathematics should be associated with physical actions to allow the child to more easily construct the meaning of the verb. When communicating with children having difficulty in mathematics, the teacher should use statements that contain only one or two ideas, and have relatively simple grammatical structures.

The Inability to Comprehend Problems

If a child is having difficulty in comprehending mathematical statements and symbols, we should extend and develop the full meaning of words being used by encouraging the child

- to verbalize the entire statement, using his own language;
- to represent the information pictorally, or to translate it as a series of actions in time, on concrete or pictorial data;
- to learn reading skill for symbols including the means for breaking up sets of symbols.

Variations of the Meanings of Words in the Mathematics Context

In order to overcome the problem of misunderstanding the variations of the words used in maths texts, the teacher should try to ensure that the student is exposed to several instances of the word, one instance at a time, and encouraged to identify the properties critical to the meaning of the word.

The teacher can also encourage the child to list characteristics of the word meaning and formulate their own definition.

Care should also be taken when choosing a maths text so as to eliminate over-exposure of too many word variables too soon.

Syntax in Mathematical Texts

The syntax of mathematical sentences is often far removed from that of natural speech. It represents a new predictive pattern for the learner to begin to use. He can be encouraged to learn this pattern by doing repeated examples of the various ways in which mathematical statements can be written, spoken or recorded, either in symbols or prose.

In order to help the learner master the spatial ordering of this language, games to develop spatial relations may be of some assistance, as well as having the added advantage of aiding concrete recording.

It is advisable for the teacher, when planning such remediation programs, to consider that the relationship between spoken language and written symbolic language is perhaps unique to maths, and therefore should be treated with the priority it deserves.

RECORDING

It is one thing to read, understand and do a maths problem, but it is another to record it. Many children have difficulty in expressing a problem using mathematical symbols and patterns, to which they are unaccustomed.

Teachers must expand children's concepts of spatial relations prior to expecting them to be able to set out mathematics problems in the conventional way, and to interpret the relations between mathematical symbols when their spatial order is rearranged.

THE APPEARANCE OF TEXTS

The appearance and variations of mathematics books is very important as it could mean the difference between a disinterested learner and one whose curiosity has been aroused.

The overall appearance of the page is important and should include such factors as simplicity, clarity and density of material. Colour to a degree, should also be included, but not to the extent that it is confusing to the student.

When choosing a maths text, the teacher should look at the readability, vocabulary, comprehension and perception level of the text, while keeping in mind the age and stage of development of his/her pupils.

There should also be some flexibility included in the text to allow for individuals to seek a new approach to a problem in which they have previously experienced failure.

John Munro gives a forthright view of the importance of language in mathematics:

> For many children, mathematics learning is akin to learning new and unfamiliar ideas in a foreign language. We must ensure that they have the opportunity to both acquire mathematical skills and understanding in a language that they understand and then learn the mathematical language describing the ideas. Learning the mathematical way of talking about an idea can only come when the child has an adequate grasp of the idea.
>
> (From J Munro, Reading Symbolic Mathematics Statements)

CASE STUDIES

When ten year old R. was asked to add numbers together her instant reply was "does that mean plus?". Similarly, if asked to subtract numbers her question was "is that minus?". This child clearly needed to broaden her understandings of mathematical language.

J. aged ten, was asked to use the four digits 6,2,8,3 to show the largest number possible. Immediately prior to this question J. had been doing additions of pairs of two digit numbers, and so answered the former question as 62 + 83. The teacher's initial reaction that J. had a poor concept of place value had to be altered when J. produced this arrangement of digits. That is, children may hear what we ask, but not receive the meaning we intend.

SOCIOECONOMIC VARIABLES AND MATHEMATICS

The child's experiences during his infancy and preschool years determine how well that child will adapt and progress in the school situation. If a child starts school without having enjoyed a rich and varied range of experiences, and if he has not been able to communicate with people who use language with flexibility and imagination, then these opportunities will have to be made available by the school. The provision of these opportunities must be a high priority of the school since they provide the foundations for future development.

Learning starts within the environment of the family, with parental attitudes important in establishing a child's values and influencing the potential of future successes. Parents who are well educated themselves, or at least regard education highly are likely to encourage their children to be confident in their potential to learn and succeed.

Children's development will be encouraged if parents support their child in his exploration of the environment.

That is, parents need to encourage children to wonder at all that is happening around them, to talk about their experiences and to allow them to have experiences that satisfy and extend their curiosity. A parent who reprimands a child for being messy or for doing things where the child may hurt himself may not be doing the child a favour - parents need to set limits for their children's behaviour but an overly conscientious parent may be handicapping their child by preventing them from participating in worthwhile learning experiences, as instanced by the following examples.

> A child had been brought up in a rural situation where he had very little contact with other people. As a five year old the child was barely able to communicate with his parents. At school he was unable to converse either with the teacher or with other children - his position was intolerable. Two years of intensive speech therapy significantly improved his communicative ability, but his parents' desire for a quiet, retiring and isolated life had seriously handicapped all his future learning.

Children from low income families or from one parent families are likely to have a more limited range of experiences than children from higher income families, or families where both parents are present. The availability of money to be spent providing worthwhile learning experiences, and the availability of parents to listen to children expressing themselves are significant elements in predicting success at school. The pre-occupation of low income families with survival and the understandable tiredness of sole parents attempting to fulfill the role of two parents means that education and school may unintentionally be seen by the child as of little importance.

The quantity and quality of newspapers and magazines a family buys, the types of television programs they watch and their leisure time activities all influence the values and attitudes of children. Lower income groups spend little on reading material and generally make little use of library facilities: children in this situation cannot help but adopt the attitudes implicit in this situation, that reading and education are unimportant.

SEX DIFFERENCES AND MATHEMATICS

Most studies agree that there are sex differences in mathematical performance from pre-school right up to and past tertiary level. The majority of findings stress that sex differences in mathematical performance are insignificant in children to about Grade 6 or 7. Despite extensive research no innate differences can be found except for minor deviations in performance, not so much ability. The main part of significant sex differences focus upon:

(a) girls excel boys in arithmetic reasoning, numeracy, counting, categorizing and memory recall;

(b) while boys tend to outperform girls in spatial abilities and general practical maths application.

In short, girls know how to do maths, but boys are better able to use maths. These differences are not striking enough to warrant serious distinction, but they exist. This does not imply that one sex possesses less brains to perform mathematics as efficiently as the other, rather it is a question of environmental influences at work that inhibit girls to fully develop their mathematical abilities.

Girls at primary school age tend to be slightly ahead of boys in language development and arithmetic reasoning. At Grade 6 the boys' early lag in intellectual development had been overcome and they should at least be equal to girls in standardized achievement tests. When these tests are adjusted for intelligence the results were often better for boys, who have become fully linguistic and are catching up on arithmetic ability. Boys are better in spatial abilities throughout primary school and have a better theoretic understanding of mathematics, so have a growing lead over girls as they enter the secondary school years. A disproportionally

low number of girls do math related courses in secondary and tertiary levels and few enter professions of a very mathematical nature. We have few women mathematicians, scientists or technicians, despite the drive for sexual equality and equal opportunities for women.

Girls can do mathematics, but they do not: why? There is no simple answer to this question.

Boys are usually more positively reinforced towards mathematics than girls, and from an early age. Games, books and toys tend to have boys building, exploring and experimenting, whereas girls activities are more passive. By the time children start school many attitudes and values are already established, and numerous school books and activities continue this discrimination between the sexes. Teachers need to take more care when teaching mathematics to girls so as not to aggravate the already one sided situation. For example, girls who become primary school teachers frequently believe themselves to be incompetent in mathematics, thus setting up an unfortunate circular process.

In general, girls appear to be less aware than boys of the importance and usefulness of mathematical competence for later career prospects. Without a mathematics background girls are excluding themselves from the fields of science and technology. The situation where girls see little relevance in studying mathematics is made more acute if teachers, the family, counsellors and society in general reflect these views. Girls do not possess an innate lack of motivation towards mathematics, the reality is that their motivation is dampened by the whole range of factors described above; and labelling mathematics as a masculine subject has a negative effect upon all but the most determined girls.

CASE STUDIES

Eleven year old R. has quite a problem with Mum and Dad's clearly defined sex roles and their attitudes towards her schooling. R's teacher often praises R. to her mother, who generally laughs heartily and changes the subject: the father's disinterest is complete.

When R. had designed a house, as part of the mathematics program, it was placed on the school display board. The mother visited the school shortly after, and on seeing the design laughed heartily in front of R.; and asked to be able to take the design home to give the rest of the family a good laugh.